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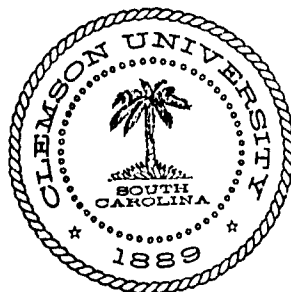
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6 A SEQUENTIAL PROCEDURE FOR SELECTING
THE MOST PROBABLE MULTINOMIAL EVENT.

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A SEQUENTIAL PROCEDURE FOR SELECTING
THE MOST PROBABLE MULTINOMIAL EVENT

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ABSTRACT

This paper deals with a sequential sampling procedure for selecting from a given multinomial distribution with K cells, the cell with the largest probability of occurrence. Observations being taken sequentially from the given distribution, the sampling is terminated when the largest count in any cell is equal to N or when the difference between the largest and the next largest cell counts is equal to r , where r and N are given positive integers. The cell with the largest count on the termination of sampling is selected as the most probable event. The probability of a correct selection and the expected total number of observations are given.

Key words: Multinomial Distribution; Ranking and Selection.

AMS Classification: 62F07

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1. Introduction. Consider a multinomial distribution with $k \geq 2$ cells and the associated probabilities p_1, \dots, p_k where $\sum_{i=1}^k p_i = 1$. Let $p_{[1]} \leq p_{[2]} \leq \dots \leq p_{[k]}$ denote the ordered values of p_1, \dots, p_k . Various sampling rules have been considered in the literature for selecting the most probable event, that is, the event associated with $p_{[k]}$. Bechhofer, et al. (1959) have considered a fixed sample procedure. Cacoullos and Sobel (1966) and Alam (1971) have considered sequential sampling procedures. The sampling procedure of Cacoullos and Sobel is called inverse sampling in which the observations being taken sequentially from the given distribution, the sampling is terminated when the count in any cell reaches a given positive integer N . Clearly, the sample size is bounded above by $KN - K + 1$. In Alam's procedure the sampling is terminated when the difference between the largest and the next largest cell count is equal to a given positive integer r . Alam's procedure seems to be more intuitive than the procedure of Cacoullos and Sobel. In this paper we consider a new sampling procedure which is a combination of the sampling rules of Alam and Cacoullos and Sobel. In this procedure the sampling is terminated when the largest cell count reaches N or the difference between the largest and the next largest cell count is equal to $r \leq N$, whichever event occurs first. We call the new procedure R . As in the case of inverse sampling, the sample size for R is less than or equal to $KN - K + 1$. Thus the procedure R combines the intuitive approach of Alam with the bounded sample size approach of Cacoullos and Sobel.

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In each of the above procedures the cell with the largest count at the termination of sampling is selected as the most probable event. However, in the fixed sample procedure of Bechhofer a randomization is necessary for selecting the most probable event in case two or more cells are tied for the largest count.

We consider the problem of selecting the most probable event from the classical approach of ranking and selection, as follows: Let $\underline{p} = (p_1, \dots, p_K)$. Given two numbers $P^* (\frac{1}{K} < P^* < 1)$ and $\theta > 1$, it is required to determine values of r and N for the procedure R , such that the probability of a correct selection (PCS) is at least as large as P^* for all values of \underline{p} for which

$$(p_{[K]}/p_{[K-1]}) \geq \theta. \quad (1.1)$$

A value of \underline{p} which minimizes the PCS, given (1.1), is called the least favorable configuration (LFC). It was shown by Kesten and Morse (1959) that the LFC for the fixed sample procedure of Bechhofer is given by

$$\begin{aligned} p_{[i]} &= (\theta + K - 1)^{-1}, \quad i=1, \dots, K-1 \\ p_{[K]} &= \theta(\theta + K - 1)^{-1}. \end{aligned} \quad (1.2)$$

Cacoullos and Sobel have shown in their paper that the LFC for the inverse sampling procedure also is given by (1.2). For $K=2$ it is clear that the LFC for the procedure R is given by $(p_{[2]}/p_{[1]}) = \theta$. But for $K>2$ it is not possible to determine

LFC analytically, since the expression for the PCS is mathematically intractable, as is the case with Alam's procedure.

However, it has been found through empirical evaluation of the PCS that the LFC is given by (1.2) for $K=3$. Therefore, we conjecture that this result holds for all $K \geq 2$.

In the next section we consider the probability of a correct selection and the expected sample size for the procedure R. We also compare the procedure R with the three selection procedures mentioned above.

Gupta and Nagel (1967) have considered the problem of selecting the most probable multinomial event from another approach, namely subset selection, in which it is required to select a subset of K events which includes the most probable event.

2. Main results. The probability of a correct selection and the expected sample size for the procedure R, given the values of r and N , are derived as follows: the multinomial probability is given by

$$g(x_1, \dots, x_K) = n! \prod_{i=1}^K \frac{p_i^{x_i}}{x_i!} \quad (2.1)$$

where $\sum_{i=1}^K x_i = n$. Let $x'_1 \leq x'_2 \leq \dots \leq x'_K$ denote the ordered values of the components of $x = (x_1, \dots, x_K)$ and let $x \in A_n$ if either $x'_K = N$ or $x'_K - x'_{K-1} = r$. For $x \in A_n$ let $f_n(x_1, \dots, x_K)$ denote the total number of ways of distributing $n = \sum_{i=1}^K x_i$ different balls into K cells one at a time, such that, at any stage in time except at stage n , the largest cell count is less than N

and the difference between the largest and next largest cell count is less than r . At stage n there are x_i balls in the i th cell, $i=1, \dots, K$. We have

$$PCS = \sum_{n=r}^{KN-K+1} \sum_n f_n(x_1, \dots, x_K) p_{[1]}^{x_1} \dots p_{[K]}^{x_K} \quad (2.2)$$

where \sum_n denotes the summation over the set of vectors $x \in A_n$ for which $x_K = x'_K$. The expected sample size S is given by

$$E(S) = \sum_{n=r}^{KN-K+1} \sum_{x \in A_n} n f_n(x_1, \dots, x_K) p_1^{x_1} \dots p_K^{x_K} \quad (2.3)$$

A recursive relation for computing $f_n(x_1, \dots, x_K)$ is given by

$$f_n(x_1, \dots, x_K) = g_n(x_1, \dots, x_K) - \sum_{m=r}^{n-1} \sum_{y \in B_m(x)} f_m(y_1, \dots, y_K) g_{n-m}(x_1 - y_1, \dots, x_K - y_K)$$

where $B_m(x)$ denotes the set of vectors $y = (y_1, \dots, y_K) \in A_m$ for which $y_i \leq x_i$, $i=1, \dots, K$. The above relation is satisfactory for computation only when K and n are small. We have used it to compute the values of PCS and $E(S)$ for $K=3$. We have applied Monte Carlo method, using 1000 trials, to compute those values for $K>3$.

For $K=2$ it is fairly easy to derive exact expressions for the values of PCS and $E(S)$, as shown below. Consider a random walk generated by a sequence of i.i.d. random variables $\{Z_n\}$ for which $P\{Z_n=1\} = p$ and $P\{Z_n=-1\} = q$ where $p=p_{[2]}$ and $q=p_{[1]}$, corresponding to the sampling procedure R . Let $S_n = \sum_{i=1}^n Z_i$. Suppose that the sampling is stopped at stage n . Then $S_n = \pm r$ or $S_n = \pm(2N-n)$. We observe that $S_n = \pm r$ for $r \leq n \leq 2N-r$ and $S_n = \pm(2N-n)$ for $2N-r < n \leq 2N-1$.

Let $A(n, c; a, -b)$ denote the number of paths in the random walk, leading to $S_n = c$ that do not meet the line $y = a$ or $y = -b$ where $-b < c < a$ is given by (see e.g. Billingsley (1968), Formula (11.5), p. 78)

$$A(n, c; a, -b) = L_{n, c} - \sum_{K=1}^{\infty} (L_{n, 2(K-1)(a+b)+2a-c} - L_{n, 2K(a+b)+c} + L_{n, 2(K-1)(a+b)+2b+c} - L_{n, 2K(a+b)-c}) \quad (2.4)$$

where $L_{n, c} = \binom{n}{\frac{n+c}{2}}$ denotes the total number of paths leading to $S_n = c$. From (2.4) we have after simplification

$$A(n-1, r-1; r, -r) = \binom{n-1}{\frac{n+r-2}{2}} - \sum_{K=1}^{\lfloor \frac{n+3r-2}{4r} \rfloor} \left(\binom{n-1}{\frac{(4K-3)r+n}{2}} + \binom{n-1}{\frac{(4K-1)r+n-2}{2}} - \binom{n-1}{\frac{(4K-1)r+n}{2}} - \binom{n-1}{\frac{(4K+1)r+n-2}{2}} \right) \quad (2.5)$$

$$A(n-1, 2N-n-1; r, -r) = \binom{n-1}{N-1} - \sum_{K=1}^{\lfloor \frac{N+r-1}{2r} \rfloor} \left(\binom{n-1}{(2K-1)r-N+n} + \binom{n-1}{(2K-1)r+N-1} - \binom{n-1}{2Kr+N-1} - \binom{n-1}{2Kr-N+n} \right) \quad (2.6)$$

where $\lfloor y \rfloor$ denotes the greatest integer less than or equal to y and $\binom{n}{y} = 0$ if $y > n$. Associating the random walk with the procedure R we derive the following result:

$$\begin{aligned}
 PCS &= \sum_{n=r, r+2, \dots, 2N-r} A(n-1, r-1; r, -r) p^{\frac{n+r}{2}} q^{\frac{n-r}{2}} \\
 &+ \sum_{n=2N-r+1}^{2N-1} A(n-1, 2N-n-1; r, -r) p^N q^{n-N} \quad (2.7)
 \end{aligned}$$

$$\begin{aligned}
 E(S) &= \sum_{n=r, r+2, \dots, 2N-r} n A(n-1, r-1; r, -r) (p^{\frac{n+r}{2}} q^{\frac{n-r}{2}} + p^{\frac{n-r}{2}} q^{\frac{n+r}{2}}) \\
 &+ \sum_{n=2N-r+1}^{2N-1} n A(n-1, 2N-n-1; r, -r) (p^N q^{n-N} + p^{N-n} q^N) \quad (2.8)
 \end{aligned}$$

The formulas for PCS and E(S) are simplified for $r=2, 3$ and $N-1$, as given below.

$K=2, r=2$.

$$\begin{aligned}
 PCS &= \frac{p^2 (1 - (2pq)^{N-1})}{1 - 2pq} + 2^{N-1} p^N q^{N-1} \\
 E(S) &= \sum_{n=2, 4, \dots, 2N-2} n \cdot 2^{\frac{n-2}{2}} (p^{\frac{n+2}{2}} q^{\frac{n-2}{2}} + p^{\frac{n-2}{2}} q^{\frac{n+2}{2}}) \\
 &+ (2N-1) (p^N q^{2N-1} + p^{2N-1} q^N).
 \end{aligned}$$

$K=2, r=3$.

$$\begin{aligned}
 PCS &= \frac{p^3 (1 - (3pq)^{N-2})}{1 - 3pq} + 3^{N-2} p^N q^{N-2} (1 + 2q) \\
 E(S) &= \sum_{n=3, 5, \dots, 2N-3} n \cdot 3^{\frac{n-3}{2}} (p^{\frac{n+3}{2}} q^{\frac{n-3}{2}} + p^{\frac{n-3}{2}} q^{\frac{n+3}{2}}) \\
 &+ 3^{N-2} (p^N q^{N-2} + p^{N-2} q^N) + 2 \cdot 3^{N-2} (p^N q^{N-1} + p^{N-1} q^N).
 \end{aligned}$$

$K=2, r=N-1$.

$$\begin{aligned}
 PCS &= p^{N-1} + (N-1) p^N q + \sum_{j=2}^{N-2} ((\binom{N+j-1}{N-1} - 1) p^N q^j) \\
 &+ ((\binom{2N-2}{N-1} - 2) p^N q^{N-1}) \\
 &= I_p(N, N) - p^{N-1} q^{N-1} (2p-1)
 \end{aligned}$$

$$\begin{aligned}
E(S) = & (N-1)(p^{N-1} + q^{N-1}) + (N^2-1)(p^N q + p q^N) \\
& + \sum_{j=2}^{N-2} (N+j) \binom{N+j-1}{N-1} p^N q^j \\
& + (2N-1) \left(\binom{2N-2}{N-1} - 2 \right) p^N q^{N-1}
\end{aligned}$$

where

$$I_p(m, n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)} \int_0^p x^{m-1} (1-x)^{n-1} dx$$

denotes the incomplete beta function.

Table I gives the values of PCS and $E(S)$ for the least favorable configuration given by (1.2) and specified values of r, N and ϵ . The values of PCS and $E(S)$ are given in that order within parentheses in the table. The figures for $K > 3$ were computed from the results of a simulation study based on 1000 repetitions of the sampling process.

In the classical formulation of the selection problem it is required to determine the values of (r, N) for which $PCS \geq P^*$, given $p_{[K]}/p_{[K-1]} \geq \epsilon$, where P^* and ϵ are given numbers. For each r we select the smallest value of N for which $PCS \geq P^*$. There may be several pairs of values of (r, N) for which $PCS \geq P^*$. However, it has been found empirically that the pair corresponding to the smallest value of r leads to the smallest value of $E(S)$. Therefore, we select this pair. The following table gives values of such pairs (r, N) and $E(S)$ for $P^* = .75, .90, \epsilon = 1.6, 2.0, 2.4, 3.0$ $K=2(1)6$. The figures given in the table for $K \geq 4$ are based on Monte Carlo results.

Values of $(r, N, E(S))$ for $PCS \geq P^*$ $P^* = .75$

K:	2	3	4	5	6
$\theta = 1.6$	(3,5,5.96)	(4,12,17.19)	(4,19,30.31)	(4,23,41.08)	(5,25,66.34)
2.0	(2,3,3.09)	(3,6,8.44)	(3,9,13.56)	(3,11,18.76)	(3,16,23.84)
2.4	(2,2,2.42)	(2,5,4.70)	(3,5,9.07)	(3,6,12.57)	(3,6,14.53)
3.0	(1,1, 1)	(2,3,3.68)	(2,5,5.58)	(2,5,6.88)	(2,8,8.30)

 $P^* = .90$

$\theta = 1.6$	(5,21,17.00)	(6,30,34.77)	(10,28,61.95)	(8,31,84.09)	(8,36,112.20)
2.0	(4,8,8.90)	(4,15,15.01)	(4,19,23.39)	(4,21,30.49)	(5,20,47.87)
2.4	(3,6,5.72)	(3,20,8.90)	(4,9,15.91)	(4,11,21.58)	(4,13,27.61)
3.0	(2,14,3.20)	(3,5,6.44)	(3,8,10.02)	(3,8,12.84)	(3,9,14.80)

To compare the procedure R with the three procedures mentioned in the previous section, the following table shows the minimum expected sample size rounded to the next largest integer for the four procedures corresponding to the least favorable configuration given by (1.1) for $\theta = 1.6, 2.0, 3.0$ for which $PCS \geq P^* = .75, .90$. We denote in the table the sample size for the fixed sample procedure of Bechhofer et. al. [2] by F and the expected sample size for the sequential sampling procedures of Cacoullos and Sobel [4] and Alam [1] by $E(I)$ and $E(A)$, respectively. The values of F and $E(I)$ are reproduced from Table II of Alam [1]. Note that the procedure R is the same as Alam's procedure when $N \rightarrow \infty$. It is seen from the table that

$$F \geq E(I) \geq E(A) \geq E(S).$$

Values of $(F, E(I), E(A), E(S))$ $P^* = .75$

K:	2	3	4
$\theta = 1.6$	(9, 8, 8, 6)	(26, 23, 21, 18)	(46, 46, 31, 31)
2.0	(5, 4, 4, 4)	(12, 11, 11, 9)	(20, 19, 16, 14)
3.0	(1, 1, 1, 1)	(5, 5, 5, 4)	(8, 8, 6, 6)

 $P^* = .90$

$\theta = 1.6$	(31, 26, 18, 17)	(64, 55, 37, 35)	(98, 87, 66, 62)
2.0	(15, 12, 11, 9)	(28, 26, 16, 15)	(56, 36, 26, 24)
3.0	(7, 6, 4, 4)	(11, 9, 8, 7)	(16, 14, 11, 11)

REFERENCES

- [1] Alam, K. (1971). On selecting the most probable category. Technometrics (13) 843-850.
- [2] Béchhofer, R., Elmaghrabi, S. and Morse, N. (1959). A single sample multiple-decision procedure for selecting the multinomial event which has the highest probability. Ann. Math. Statist. (30) 102-119.
- [3] Billingsley, P. (1968). Convergence of Probability Measures, Wiley Series in Probability and Mathematical Statistics.
- [4] Cacoullos, T. and Sobel, M. (1966). An inverse sampling procedure for selecting the most probable event in a multinomial distribution. Proceedings of the International Symposium on Multivariate Analysis, pp. 423-455, Academic Press, Inc. New York.
- [5] Gupta, S. S. and Nagel, K. (1967). On selection and ranking procedures and order statistics from the multinomial distribution. Sankhyā. Series B (29) 1-17.
- [6] Kesten, H. and Morse, N. (1956). A property of the multinomial population. Ann. Math. Statist. (30) 120-127.

Table I-Probability of correct selection and expected sample size

K=2

 $\theta = 1.2$

N	10	15	20	25
r=2	(.590, 4.0)	(.590, 4.0)	(.590, 4.0)	(.590, 4.0)
4	(.647, 11.8)	(.662, 13.8)	(.669, 14.7)	(.672, 15.1)
6	(.656, 15.4)	(.686, 21.0)	(.706, 24.8)	(.720, 27.4)
8	(.656, 16.3)	(.689, 24.1)	(.715, 30.5)	(.735, 35.6)

 $\theta = 1.6$

2	(.719, 3.8)	(.719, 3.8)	(.719, 3.8)	(.719, 3.8)
4	(.832, 10.8)	(.855, 12.1)	(.863, 12.5)	(.866, 12.7)
6	(.848, 14.3)	(.892, 18.4)	(.916, 20.5)	(.929, 21.7)
8	(.849, 15.4)	(.898, 21.8)	(.927, 26.0)	(.946, 28.6)
10	(.849, 15.6)	(.898, 23.3)	(.929, 29.4)	(.950, 33.6)

 $\theta = 2.0$

N	6	9	12	15
r=2	(.800, 3.6)	(.800, 3.6)	(.800, 3.6)	(.800, 3.6)
3	(.860, 6.1)	(.880, 6.7)	(.886, 6.9)	(.888, 7.0)
4	(.875, 7.7)	(.912, 9.3)	(.929, 10.0)	(.936, 10.3)
5	(.878, 8.4)	(.922, 11.1)	(.945, 12.5)	(.957, 13.3)
6	(.878, 8.6)	(.924, 12.3)	(.950, 14.5)	(.965, 15.8)

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$\theta = 2.4$

N	6	9	12	15
2	(.850, 3.4)	(.852, 3.4)	(.852, 3.4)	(.852, 3.4)
3	(.911, 5.7)	(.927, 6.2)	(.931, 6.3)	(.932, 6.3)
4	(.926, 7.2)	(.955, 8.5)	(.965, 8.9)	(.969, 9.1)
5	(.928, 8.0)	(.962, 10.2)	(.978, 11.2)	(.983, 11.5)
6	(.928, 8.3)	(.964, 11.5)	(.981, 13.0)	(.988, 13.8)

$\theta = 3.0$

2	(.899, 3.2)	(.899, 3.2)	(.899, 3.2)	(.900, 3.2)
3	(.952, 5.3)	(.962, 5.5)	(.964, 5.6)	(.964, 5.6)
4	(.964, 6.7)	(.982, 7.5)	(.986, 7.7)	(.987, 7.8)
5	(.966, 7.6)	(.986, 9.2)	(.993, 9.7)	(.995, 9.8)
6	(.966, 7.9)	(.987, 10.5)	(.995, 11.5)	(.997, 11.8)

Table I (Continued)

K=3

 $\theta = 1.2$

N	10	15	20	25
r=2	(.430, 6.1)	(.430, 6.1)	(.430, 6.1)	(.430, 6.1)
4	(.492, 17.6)	(.517, 21.9)	(.529, 24.1)	(.538, 25.1)
6	(.499, 21.5)	(.537, 31.2)	(.566, 38.6)	(.587, 44.2)
8	(.499, 22.1)	(.539, 34.3)	(.572, 45.1)	(.600, 54.5)

 $\theta = 1.6$

2	(.588, 5.7)	(.588, 5.7)	(.588, 5.7)	(.588, 5.7)
4	(.735, 15.8)	(.779, 18.5)	(.797, 19.6)	(.804, 20.0)
6	(.749, 19.8)	(.820, 26.8)	(.863, 31.1)	(.888, 33.4)
8	(.750, 20.8)	(.825, 30.7)	(.874, 37.9)	(.907, 42.7)

 $\theta = 2.0$

N	6	10	15	20
2	(.691, 5.2)	(.697, 5.3)	(.697, 5.3)	(.697, 5.3)
3	(.773, 8.4)	(.820, 10.0)	(.833, 10.4)	(.835, 10.4)
4	(.789, 10.2)	(.867, 13.6)	(.901, 15.0)	(.911, 15.4)
5	(.791, 10.9)	(.881, 16.1)	(.929, 18.8)	(.947, 19.7)
6	(.791, 11.1)	(.883, 17.7)	(.938, 21.9)	(.962, 23.5)

 $\theta = 2.4$

2	(.766, 4.8)	(.771, 4.9)	(.771, 4.9)	(.771, 4.9)
3	(.853, 7.7)	(.891, 8.7)	(.898, 8.9)	(.899, 8.9)
4	(.869, 9.4)	(.933, 11.7)	(.953, 12.3)	(.957, 12.4)
5	(.871, 10.2)	(.944, 14.0)	(.973, 15.2)	(.980, 15.4)
6	(.871, 10.5)	(.946, 15.7)	(.979, 17.8)	(.989, 18.2)

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$$\theta = 3.0$$

N	6	10	15	20
2	(.839, 4.4)	(.842, 4.5)	(.842, 4.5)	(.842, 4.5)
3	(.919, 6.8)	(.943, 7.3)	(.946, 7.4)	(.946, 7.4)
4	(.934, 8.4)	(.973, 9.6)	(.981, 9.7)	(.982, 9.7)
5	(.936, 9.4)	(.981, 11.5)	(.992, 11.7)	(.994, 11.7)
6	(.936, 9.8)	(.982, 13.2)	(.995, 13.6)	(.998, 13.6)

Table I (Continued)

K=4

 $\theta = 1.2$

N	10	15	20	25
r=2	(.360, 8.14)	(.360, 8.20)	(.360, 8.20)	(.360, 8.20)
4	(.417, 23.80)	(.460, 31.61)	(.479, 35.65)	(.483, 37.83)
6	(.426, 27.39)	(.460, 41.85)	(.490, 53.87)	(.509, 63.35)
8	(.426, 27.69)	(.461, 44.11)	(.488, 59.98)	(.517, 74.42)
10	(.426, 27.70)	(.461, 44.44)	(.488, 61.28)	(.516, 77.85)

 $\theta = 1.6$

2	(.523, 7.32)	(.521, 7.86)	(.523, 7.75)	(.501, 7.58)
4	(.632, 21.99)	(.731, 27.25)	(.762, 30.10)	(.771, 30.48)
6	(.670, 25.16)	(.769, 37.06)	(.814, 45.28)	(.840, 50.80)
8	(.679, 26.19)	(.773, 40.24)	(.847, 51.59)	(.878, 62.41)
10	(.675, 25.80)	(.779, 41.21)	(.840, 54.73)	(.865, 66.13)

 $\theta = 2.0$

2	(.643, 7.17)	(.625, 7.28)	(.626, 7.01)	(.666, 7.35)
3	(.765, 14.41)	(.806, 15.33)	(.798, 15.87)	(.798, 16.05)
4	(.831, 19.33)	(.888, 22.65)	(.900, 23.42)	(.894, 23.84)
5	(.838, 22.05)	(.929, 27.66)	(.924, 31.14)	(.942, 31.00)
6	(.854, 23.02)	(.924, 31.83)	(.951, 35.16)	(.965, 38.18)

(16)

$\theta = 2.4$

N	10	15	20	25
r=2	(.700,6.48)	(.723,6.57)	(.723,6.71)	(.718,6.48)
3	(.865,12.35)	(.879,13.00)	(.878,13.73)	(.871,13.36)
4	(.925,17.34)	(.949,18.77)	(.945,19.51)	(.953,18.73)
5	(.931,19.77)	(.966,23.87)	(.977,23.79)	(.982,24.99)
6	(.930,20.95)	(.968,26.56)	(.983,28.43)	(.985,29.34)

$\theta = 3.0$

2	(.805,5.81)	(.808,6.04)	(.816,5.66)	(.820,5.79)
3	(.917,10.52)	(.928,10.78)	(.937,10.86)	(.935,11.22)
4	(.959,13.71)	(.980,14.80)	(.969,15.19)	(.976,14.80)
5	(.974,16.55)	(.992,18.48)	(.992,19.27)	(.994,19.08)
6	(.966,18.43)	(.994,21.64)	(.996,21.91)	(1.000,22.49)

Table I (Continued)

K=5

N	10	15	20	25
$\theta = 1.2$				
r=2	(.290,11.04)	(.313,10.37)	(.319,10.88)	(.309,10.32)
4	(.351,29.25)	(.366,40.54)	(.371,46.82)	(.387,52.23)
6	(.345,32.50)	(.366,51.26)	(.436,66.49)	(.453,80.13)
8	(.357,32.76)	(.370,53.60)	(.435,72.40)	(.410,91.60)
10	(.355,33.00)	(.371,53.20)	(.432,73.88)	(.455,95.02)
$\theta = 1.6$				
2	(.459,9.79)	(.439,9.95)	(.446,10.31)	(.466,10.38)
4	(.599,27.60)	(.661,34.60)	(.679,38.86)	(.734,41.61)
6	(.626,30.85)	(.727,46.06)	(.798,56.41)	(.837,63.70)
8	(.602,30.96)	(.749,48.83)	(.798,63.84)	(.843,76.92)
10	(.646,30.64)	(.671,49.53)	(.791,66.64)	(.844,82.71)
$\theta = 2.0$				
2	(.573,9.10)	(.590,8.93)	(.562,8.88)	(.556,9.26)
3	(.735,17.68)	(.777,19.48)	(.780,20.87)	(.771,20.14)
4	(.775,24.16)	(.864,28.94)	(.885,30.15)	(.870,30.21)
5	(.783,26.64)	(.895,34.90)	(.930,38.65)	(.950,40.77)
6	(.799,28.12)	(.904,38.99)	(.943,45.19)	(.967,47.41)
$\theta = 2.4$				
2	(.686,8.10)	(.658,8.04)	(.672,8.30)	(.671,8.25)
3	(.839,15.59)	(.854,16.66)	(.862,16.77)	(.864,17.20)
4	(.882,21.11)	(.922,23.79)	(.947,24.19)	(.944,25.00)
5	(.899,23.78)	(.960,28.75)	(.954,30.69)	(.979,31.09)
6	(.887,25.51)	(.965,32.58)	(.980,35.80)	(.985,36.65)

(18)

$$\theta = 3.0$$

N	10	15	20	25
2	(.767,6.98)	(.765,7.16)	(.765,7.19)	(.753,6.83)
3	(.913,13.63)	(.927,13.26)	(.923,13.97)	(.935,13.48)
4	(.967,17.01)	(.966,18.90)	(.981,18.65)	(.968,19.15)
5	(.960,20.04)	(.991,22.54)	(.992,23.55)	(.993,23.48)
6	(.961,22.24)	(.988,26.46)	(.998,27.39)	(.996,27.33)

Table I (Continued)

K=6

N	10	15	20	25
$\theta = 1.2$				
r=2	(.215,12.86)	(.261,12.64)	(.244,12.57)	(.241,13.09)
4	(.292,34.34)	(.321,48.58)	(.332,57.84)	(.365,61.62)
6	(.305,38.25)	(.319,60.16)	(.386,78.75)	(.414,96.94)
8	(.306,38.05)	(.339,61.69)	(.398,85.43)	(.415,107.55)
10	(.316,37.98)	(.332,62.10)	(.363,86.08)	(.409,111.95)
$\theta = 1.6$				
2	(.406,11.35)	(.432,12.36)	(.426,12.01)	(.405,12.33)
4	(.562,32.78)	(.648,41.98)	(.679,50.07)	(.711,50.39)
6	(.586,35.74)	(.660,54.49)	(.753,68.56)	(.802,77.92)
8	(.578,36.14)	(.674,57.17)	(.758,76.20)	(.818,91.37)
10	(.566,35.94)	(.666,57.22)	(.767,78.17)	(.809,96.96)
$\theta = 2.0$				
2	(.540,10.54)	(.534,10.84)	(.534,10.87)	(.536,11.25)
3	(.692,23.01)	(.743,23.59)	(.776,25.05)	(.751,24.24)
4	(.744,27.66)	(.834,35.05)	(.849,37.19)	(.889,39.12)
5	(.774,31.51)	(.869,42.61)	(.915,47.87)	(.932,48.88)
6	(.769,32.59)	(.870,46.35)	(.925,53.83)	(.951,57.40)
$\theta = 2.4$				
2	(.648,9.80)	(.677,9.95)	(.658,9.79)	(.645,9.60)
3	(.826,18.99)	(.844,20.62)	(.861,20.32)	(.844,21.52)
4	(.868,25.02)	(.923,28.91)	(.950,29.55)	(.936,28.86)
5	(.895,28.23)	(.942,34.76)	(.964,37.95)	(.964,37.65)
6	(.893,29.33)	(.945,39.46)	(.977,43.37)	(.990,44.70)

N	10	15	20	25
	$\theta = 3.0$			
2	(.754,8.32)	(.724,8.41)	(.761,8.38)	(.731,8.15)
3	(.913,15.86)	(.916,15.97)	(.913,15.63)	(.921,15.60)
4	(.953,20.42)	(.972,21.78)	(.986,22.69)	(.978,22.52)
5	(.962,23.99)	(.987,26.87)	(.993,28.13)	(.994,28.06)
6	(.946,25.52)	(.992,30.81)	(.999,31.42)	(.999,33.09)

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